

# A COMPARATIVE STUDY OF PROPORTIONAL-INTEGRAL (P-I) AND INTEGRAL-PROPORTIONAL (I-P) CONTROLLERS FOR DE MOTOR DRIVES

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A near investigation of corresponding fundamental (P-I) and basic relative (I-P) control plans for a de drive is introduced. Different qualities, for example, blunder signal handling and affectability to regulator gains, of both the plans are investigated. The reaction of both the regulators for an adjustment of speed reference and burden force is examined. The current reaction during beginning is additionally introduced. It is shown that the I-P conspire offers some particular benefits. Trial and recreation results are additionally introduced.

## Nomenclature

EN	speed error
KP	proportional gain
KI	integral gain
C1	proportional gain in current loop
Kett	gain of GTO chopper
KR	current feedback constant
KT	tachogenerator constant (used for speed feedback)
Ts	electrical torque
TL	load torque
NR	speed reference command
N	actual speed of motor

## 1. INTRODUCTION

Most de motor drives are operated as closed-loop speed-control systems. Generally, an external speed loop and an internal current loop are the most common feedback techniques for these systems (Sen 1981). A simple proportional gain in the speed loop may not be sufficient to provide a precise control on the speed of the drive. This may result in a high overshoot and also an undesirable steady-state error in speed. Therefore some kind of compensation technique has to be employed to improve the performance of the drive. Phase-lead, phase-lag and lag-lead compensation techniques are well known and are used to improve the transient and steady-state behaviour of a system (Kuo 1982). These compensation techniques are widely used and their characteristics have been fully explored. The most widely used compensation method for de motor drives is the proportional plus integral (P-I) control.

P-I control is a special case of a phase-lag compensation technique with a pole at the origin. This scheme has some good features:

- (1) Because of the integral term, the steady-state error in speed is zero, making the scheme quite accurate;
- (b) It is not necessary to use such high gains as required in

proportional-gain compensation;

(c) the P-I controller has been used in a variety of applications and the scheme has been found to be quite robust and satisfactory.

However, there are some problems with P-I control:

- (a) if a very fast response is desired, the penalty paid is a higher overshoot in the speed, which is undesirable;
- (b) the system can be designed, without any overshoot but the response to a load disturbance becomes very slow;
- (c) a very high gain cannot be used to obtain fast response, because that will result in higher overshoots.

Some industrial applications, such as robotics and rolling mills, require minimal or no overshoot in the speed and demand a fast response for a change in speed reference as well as for a load disturbance.

Recently a scheme called integral-proportional (I-P) control has been proposed (Harashima and Kondo 1982, Kayanak et al. 1983). The I-P controller tends to overcome some of the difficulties and limitations encountered with P-I controllers. However, a detailed study of the I-P controller has not yet been reported and its characteristic features have not been fully explored.

This paper presents a detailed analysis for both P-I and I-P control schemes. The characteristic features of both these schemes are studied and a comparative study is presented. Some simulation and experimental results are also presented.

## 2. P-I AND I-P TRANSFER-FUNCTION MODELS

The transient and steady-state behaviour of the two types of controllers, P-I and I-P, is studied using transfer-function models, and the system response for both control schemes for a change in reference speed and load torque is investigated.

### 2.1. P-I controller

The block diagram of the drive with the P-I controller has one outer speed loop and one inner current loop, and is shown in Fig. 1.

The speed error EN between the reference speed NR and the actual speed N of the motor is fed to the P-I controller, and Kp and KI are the proportional and integral gains of the P-I controller. The output of the P-I controller  $\epsilon_1$  acts as a

current reference command to the motor, C1 is a simple proportional gain in the current loop and Kcu is the gain of the GTO thyristor chopper, which is used as the power converter.

The current reference command  $\epsilon_1$  is clamped at a maximum value to prevent the motor current from rising to a high value during starting. For analysis, the motor electrical time constant can be neglected, since it is very small compared with the mechanical time constant of the motor.

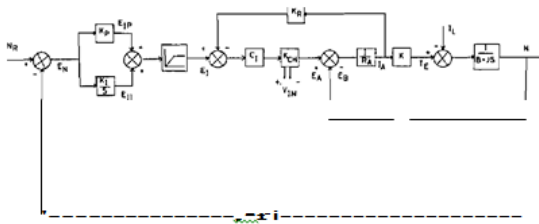


Figure 1. Block diagram of the dc drive with P-I controller.

The P-I controller has the form

$$E_1(S) = \frac{K_p S + K_i}{S} E_n(S) \quad (1)$$

This is a phase-lag type of controller with the pole at the origin and makes the steady-state error in speed zero. The transfer function between the output speed  $N$  and the reference speed  $N_r$  is given by

$$\frac{N(S)}{N_r(S)} = \frac{AK_1 + AK_p S}{K_1 S^2 + K_2 S + K_3} \quad (2)$$

where

$$\begin{aligned} A &= C_1 K_{em} K \\ K_1 &= RABTM + KRCIKCHB TM \\ K_2 &= RAB + K^2 + KRCIKCHB + AK_p \\ K_3 &= AK_r \end{aligned}$$

$K_1$  and  $K_r$  are controller gains,  $KR$  is the current feedback constant and  $R, A, B, TM$ , etc. are motor and feedback constants (these are given in the Appendix). Equation (2) shows that the P-I controller introduces a zero, and therefore a higher overshoot is expected for a step change in speed reference. This also prevents the use of high gains for  $K_1$  and  $K_r$ , an undesirable effect of the P-I controller.

The transfer function between the output speed  $N$  and the load torque  $TL$  is given by

$$\frac{N(S)}{TL(S)} = \frac{-SK_A}{K_1 S^2 + K_2 S + K_3} \quad (3)$$

where

$$K_A = R_A + K_r C_1 K_{em} \quad (4)$$

The negative sign indicates that in the transient period there will be a drop in the speed for a step increase in the load. However, in the steady state, there will not be any change in speed. This means that the steady-state error in speed is zero, which is a good feature of P-I controllers.

### 2.2. I-P controller

The block diagram of the drive with the I-P controller is shown in Fig. 2. The proportional term  $K_P$  is moved to the speed feedback path. There are three loops: one inner current loop, one speed feedback loop where output speed is compared with the reference speed and one more feedback loop through the proportional gain  $K_p$ .

The speed error  $E_N$  is fed to a pure integrator with gain  $K_I$  and the speed is fed back through a pure proportional gain  $K_p$ . The current reference signal  $E_1$  is proportional to the difference of  $E_{11}$  and  $E_{1r}$ , where  $E_{11}$  and  $E_{1r}$  are as shown in Fig. 2.

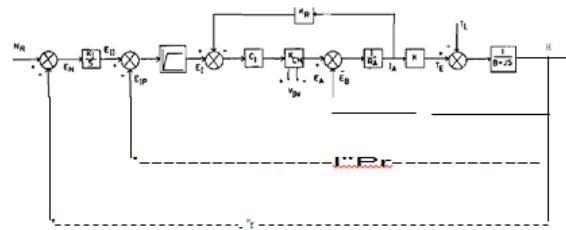


Figure 2. Block diagram of the dc drive with I-P controller.

In this case also,  $\epsilon_1$  (current reference) is clamped to clamp the armature current. The transfer function between the output speed  $N$  and the reference speed  $N_r$  is given by

$$\frac{N(S)}{N_r(S)} = \frac{AK_r}{K_1 S^2 + K_2 S + K_3} \quad (5)$$

From (2) and (5) it can be seen that the characteristic equations for both P-I and I-P are the same. However, the zero introduced by the P-I controller is absent in the case of the I-P controller, and thus the overshoot with an I-P controller is expected to be very small.

The transfer function between the output speed  $N$  and the load torque  $TL$  is given by

$$\frac{N(S)}{TL(S)} = \frac{-SK_A}{K_1 S^2 + K_2 S + K_3} \quad (6)$$

It can be seen that (3) and (6) are exactly the same, and thus the response to a load disturbance is exactly the same for both types of controller.

## 3. PROCESSING ERROR SIGNALS

Both the control schemes were simulated on a digital computer to study their behaviour. The two schemes differ in the way the speed error signal  $E_N$  is processed. The simulation study gives a better insight into the error-processing schemes and their effects on the system behaviour.

### 3.1. P-I controller

The way in which  $E_N$  changes with time, for a step change in  $N_r$ , from a standstill condition is shown in Fig. 3. The error  $E_N$  is high initially and decreases to a small value as the output speed approaches the reference speed. The output  $E_{1p}$  of the proportional gain  $K_p$  thus dominates in the initial period as shown in Fig. 4. The signal  $E_{1p}$  is clamped by the saturation of the Op-Amp. The effect of the proportional gain, however, is small once the speed error becomes small. On the other hand, the output of the integral gain cannot change instantaneously as shown in Fig. 4.  $E_{1i}$ , the output of the integral term, changes slowly during the initial period. Its effect is predominant as the motor speeds up.

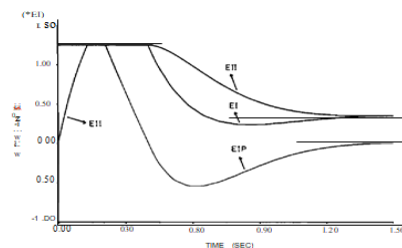


Figure 4. Variation of  $E_{1p}$ ,  $E_{1i}$ , and  $E_1$  with time for P-I control.

However, the current reference signal  $E_r$ , which is proportional to the sum of the  $\epsilon_{1p}$  and  $E_n$ , shows an instantaneous change because of the initial rapid change in  $\epsilon_{1p}$ . It can be seen in Fig. 4 that  $\epsilon_{1p}$  jumps, instantaneously, to the clamp value and stays clamped for some time before decreasing to a smaller value. The effect of this instantaneous jump in  $\epsilon_{1p}$  on the motor current is shown in Fig. 5. Initially, the chopper stays fully on for a few chopping cycles. The armature current is not instantaneously fed back because of the time delay involved in the filters used. The current therefore builds up beyond the clamped value. After some interval of time, the current comes down and stays clamped. Figure 6 shows the experimental results of the current response. The initial current jump is seen as expected and is undesirable in the P-1 controller scheme.

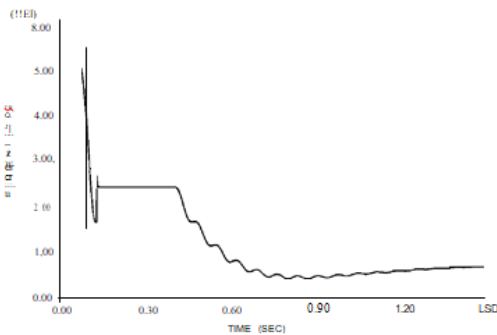


Figure 5. Starting response of armature current: simulation result for P-1 control.

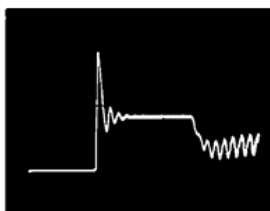


Figure 6. Starting response of armature current: experimental result for P-1 control.

### 3.2. 1-P controller

The variation of  $E_r$ , is similar to that of P-I and is shown in Fig. 7. However, the way this error signal is processed is different. The proportional gain acts on the output speed  $N$  rather than the speed error. As the output speed changes slowly, the signal  $E_r$  changes slowly with time, as illustrated in Fig. 8. This is an important difference.

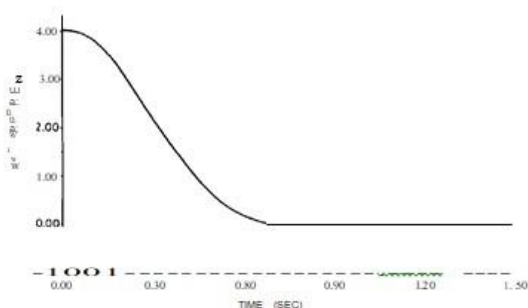


Figure 7. Variation of speed error  $E_N$  with time for 1-P control.

Even though  $E_N$  is large initially, as it is acting on a pure integrator, the signal  $E_{11}$  will not change instantaneously, as

shown in Fig. 8.

One of the most significant differences is in the generation of the current reference signal  $\epsilon_{11}$ . For the 1-P scheme  $\epsilon_{11}$  is proportional to  $(E_{11} - \epsilon_{1p})$ , and this makes a significant difference. As shown in Fig. 8, there will not be any abrupt jump in the reference signal  $\epsilon_{11}$ , as it is the difference of two signals. It can be seen from Fig. 9 that there is no jump in the actual current, which follows the current reference command. Figure 10 shows the experimental results.

The fact that the signal  $\epsilon_{11}$  is proportional to the difference of  $\epsilon_{11}$  and  $\epsilon_{1p}$  is a major advantage of this scheme. This allows the use of higher gain values. It also allows a high step command  $N_{Reven}$  from standstill conditions, without any possibility of the

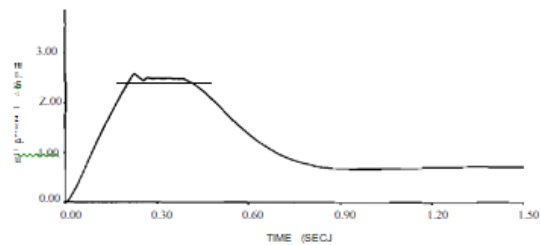


Figure 9. Starting response of armature current: simulation result for 1-P control.



Figure 10. Starting response of armature current: experimental result for 1-P control

initial current overshoot, since there will not be an abrupt change in the current reference signal  $\epsilon_{11}$ .

## 4. SPEED RESPONSE

The response time for both P-1 and 1-P schemes to a change in speed reference and load disturbance is studied in this section. It is shown that the P-1 scheme results in an overshoot in speed for a change in reference speed, while this overshoot is almost negligible for a similar rise time for the 1-P scheme.

It is also shown that the P-1 control can be designed without much overshoot. However, the speed recovery from a load disturbance, in this case, deteriorates and becomes very slow, which is undesirable. This can be explained as follows. The P-1 controller introduces a zero in the transfer function as shown in (2). However, it does not introduce a zero for a load torque disturbance as shown in (3). When a smaller overshoot in speed is desired, the damping of the system is

increased. To prevent the response from becoming too slow, the zero is adjusted such that a smaller overshoot, without sacrificing the speed of response, is obtained. However, it can be seen that by increasing damping, the response to a load disturbance will be considerably slowed down. Therefore, even if the system is designed to have no overshoot for a change in speed reference, the speed recovery from a load disturbance becomes quite slow.

4.1, Speed reference change

Figure 11 shows the simulation results of the speed response for both P-I and I-P controllers for the same damping and natural frequency. In the case of the P-I controller there is an overshoot in the speed, however, the I-P scheme shows negligible overshoot. This is also verified experimentally as shown in Figs. 12 and 13.

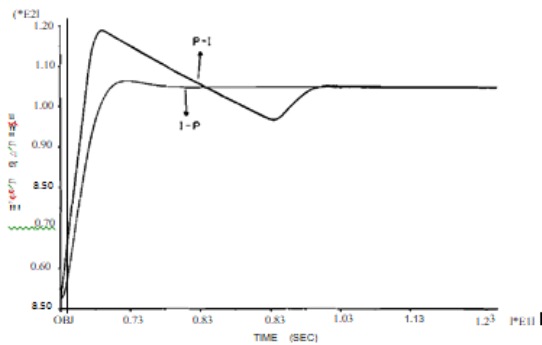


Figure 11. Response to a change in speed reference for same damping and natural frequency simulation result.

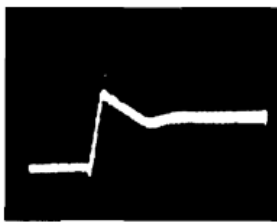


Figure 12. Response to a change in speed reference for P-I control: experimental result



Figure 13. Response to a change in speed reference for J-P control: experimental result.

These results agree with the theoretical conclusions resulting from (2) and (5) discussed in the sections on the transfer-function models.

It is shown in Fig. 14 that the P-[controller can be designed without much overshoot in speed. However, the response to a load disturbance for this case becomes very slow, as shown in Fig. 15.

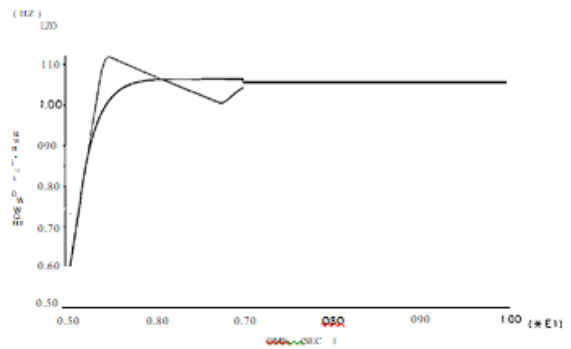


Figure 14. P-I controller response to a change in speed reference when designed for no overshoot compared with a design with overshoot.]

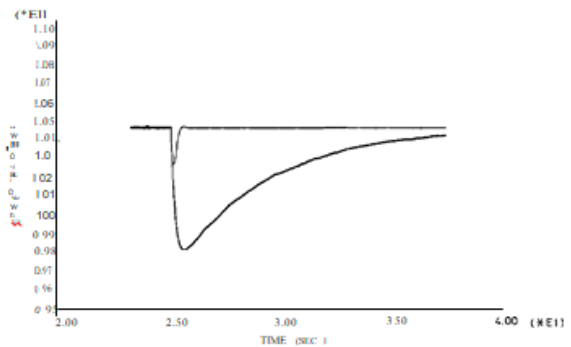


Figure 15. P-I controller response to a load disturbance for the designs in Fig. 14.

4.2. Load torque disturbance

In many industrial applications it is imperative that the system respond quickly to a load disturbance and maintain a steady constant speed. Although there is a difference in the way both P-I and I-P schemes respond to a change in speed Reference, as expected from (3) and (6), the response to a load disturbance should be the same for both.

The simulation results are shown in Fig. 16 and it can be seen that the response is very similar for both schemes. This is also verified experimentally and is shown in Figs. 17 and 18.

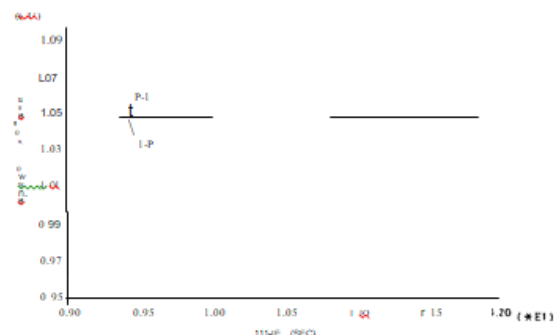


Figure 16. Response to a load disturbance for P-I and I-P controls for a similar design: simulation result.



Figure 17. Response to a load disturbance for P-I control: experimental result.

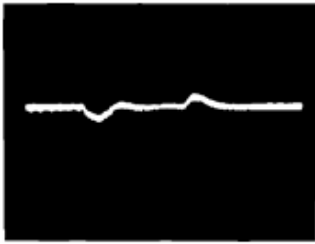


Figure 18. Response to a load disturbance for I-P control: experimental result.

4.3. Response to reductions in speed and load torque reference commands

The performance of the single quadrant chopper for an increase in the speed reference and load torque disturbance is satisfactory. However, the response to a decrease in the speed reference command or the load torque is sluggish, because of the inherent inability of a single quadrant drive to provide a negative torque. The response to a reduction in the speed reference command is shown in Fig. 19. It clearly shows how sluggish the response is.



Figure 19. Speed response for a reduction in speed reference command.

The speed and current responses due to an increase as well as decrease in the load torque are shown in Fig. 20. The current is clamped at zero during a reduction in load torque. A two-quadrant drive would have provided a negative torque and therefore would have given a faster response.

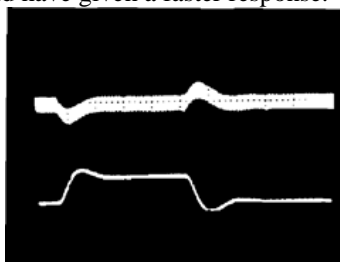


Figure 20. Response to increase and decrease in the load torque command. Top: speed, bottom: current.

S. Sensitivity to gains  $K_1$  and  $K_p$

In the P-I controller, both the proportional gain  $K_p$  and the integral gain  $K_1$  are in the forward path and act on the speed error  $e_N$ . However, in the I-P controller,  $K_p$  acts on the actual speed  $N$ , which has a higher dynamic range and  $K_1$  alone acts on  $e_N$ . In the I-P controller,  $K_p$  is in the feedback path, unlike in the P-I scheme.

The proportional gain  $K_p$  and the integral gain  $K_1$  must therefore handle higher dynamic ranges for the I-P scheme.

This is more true for the proportional gain  $K_p$ , because the actual speed  $N$  on which it acts has a wide range. On the other hand, the P-I scheme may not be very sensitive to changes in these controller gains. Some simulation results are shown in this section which verifies this behaviour.

5. CONCLUSIONS

It has been demonstrated that the I-P control scheme can give a fast response with minimal or no overshoot in speed and that the desirable feature of the P-I controller, namely a zero steady-state error in speed, is also retained in the I-P controller. However, we have also seen that the response to a load disturbance slows down considerably if the P-I controller is designed for no overshoot, which is a disadvantage.

The difference in the way the individual controller gains,  $K_p$  and  $K_1$ , act for both these schemes has been discussed in detail. The section on error signal processing explains how the current reference signal  $i^*$ , is generated and its significance for both control schemes. It has been shown, however, that the I-P scheme is more sensitive to variations in the controller gains because the controller gain  $K_p$  has to handle the actual speed instead of the speed error.

Although the P-I controller is commonly in use, the comparative study on P-I and I-P controllers presented in this paper has revealed some good features for the I-P controller, which may be useful for implementation in high-performance or precision-drive applications. The feature of no current overshoot during starting in the I-P scheme is desirable in, for example, the protection of solid state switching devices.

The inherent inability of a one-quadrant drive to provide a negative torque during speed or torque reductions has also been demonstrated. Further investigations will be carried out using a two-quadrant drive with microcomputer control.

Appendix

Machine parameters

Armature resistance  $R_A = 0.55 \Omega$

Back e.m.f. constant and electrical torque constant  $K = 0.55 \text{ N m A}^{-1}$  or  $\text{V s rad}^{-1}$

Motor mechanical inertia  $J = 0.0465 \text{ kg m}^2$

Friction coefficient  $B = 0.004 \text{ N m s rad}^{-1}$

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