# STUDY OF COMPOSITE PLATES USING ELEMENT FREE GALERKIN METHOD

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***Abstract- Mesh free methods are among the breed of numerical analysis technique that are being vigorously developed to avoid the drawbacks that traditional methods like Finite Element method possess. FEM, with half a century of passionate research behind it is versatile, time tested and trustworthy. Yet when it comes to specific areas like fracture mechanics and crack propagation, FEM has disadvantages which necessitates the need to have specialist methods dealing with such problems. The structural aspect of 'element' in FEM was found to be restrictive in nature when it comes to the implementation in such problems. Mesh free methods permit an alternative implementation based totally on nodes and devoid of the restriction of the element.***

***The current definition is approved with the specific arrangements. The reliance of the exhibition of the strategies on the boundaries concerning these techniques is broke down and the approaches to discover ideal boundaries are examined. It is discovered that EFGM gives magnificent outcomes. Nonetheless, it is reliant intensely on the boundaries like help area size. It is tracked down that the polynomial premise and weight work are the most basic boundaries and should be picked according to the underlying hypothesis utilized. The help space size, the quadrature request and nodal thickness likewise influence the outcomes fundamentally.***

***Keywords- FEM, Structural, Mesh free methods, EFGM.***

## OBJECTIVES

The objective of this work is to study the response of composite plates under static loading conditions using meshfree methods. This study aims to investigate the optimal parameter settings for obtaining accurate results using meshfree methods.

## SCOPE OF THESTUDY

The present work deals with the application of meshfree methods to the static analysis and plane stress analysis of isotropic plates and composite plates. The scope of the work is-

- To develop Element free Galerkin based algorithms found on the relevant structural theories.

## RESULTS AND DISCUSSION

## OVERVIEW

The EFGM was applied first to standard plane stress problems. EFGM was applied later to isotropic plates and then to laminate plates. The results provided here are also in that order.

## ANALYSIS OF PLANE STRESSCONDITION

The plane stress problem considered is as follows. The plate is a rectangular plate with a side fixed. The other side has traction acting on it. The meshing used in the EFG formulation is rectangular. The plane stress problem is less sensitive to the different parameters involved in the EFGM formulation. Hence the parametric study of plane stress case is not presented here. The optimal 'd' value- d being the number of times the support domain is bigger in a direction than the distance between two nodes- was found after a brief study and the displacement results extracted at that optimal 'd' value are presented here.

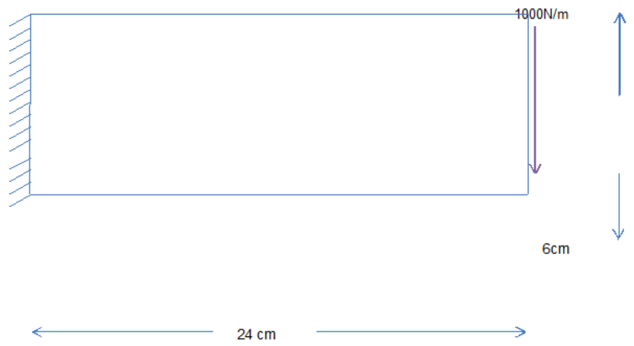


Fig 5.1. The geometry and loading of the plate

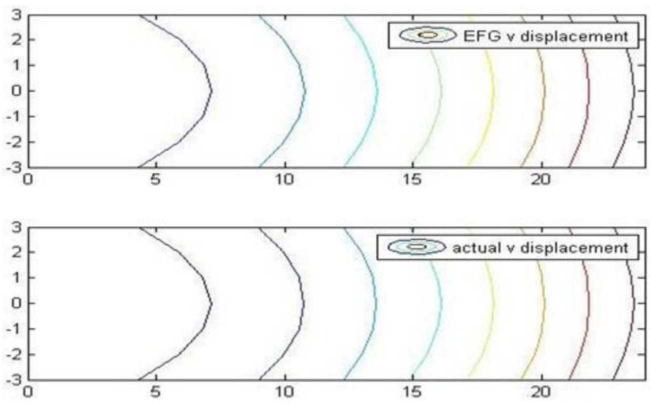


Fig 5.2a.Vertical displacement- EFG result and actual value.

  

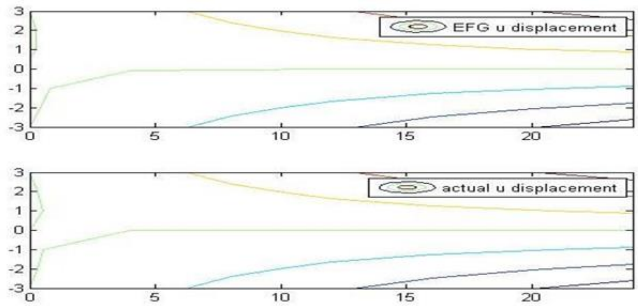


Fig. 5.2b.Horizontal displacement- EFGM results and exact results.

The figures 5.2 show the displacement in the two in-plane directions. The results are matched with the results in Timoshenko [1970]. Different scales of discretization were attempted and the results are presented in Table 5.1. An error norm has been defined here for the purpose of assessing the efficiency of the algorithm.

Error norm =

$$\sqrt{\{0.5*\int_{\Omega}(e^{num}-e^{ex})^{T}(Dmat)(e^{num}-e^{ex})\}}$$

| Number of nodes | Error Norm |
|---|---|
| 65 | 0.0260 |
| 121 | 0.0246 |
| 175 | 0.0146 |
| 279 | 0.0116 |
| 637 | 0.0076 |

Table 5.1 Convergence of error with increase in the number of nodeS

Thus the EFGM successfully analyzed the plate for plane stress condition. The error norms are seen to decrease with the fineness in discretization. The results show that the method worked well for the plane stress analysis. In addition to this problem, a laminate was also analyzed for the plane stress condition. The problem statement is as follows. A laminate composite plate of dimensions 30cm x 30 cm and thickness 1 cm was taken. It was analyzed for plane stress condition under a parabolic traction. The formulation was same as before. There were 4 layers of equal thickness in the laminate. The orientation was [0, 90]s. Few results are given here. The displacement contour is given below. Difficulties were observed in the establishment of the displacement continuity across the lamina. Different approaches have been applied in literature to solve this. One such technique is the implementation of truncated shape functions which would get restricted by the boundary of the layer. The boundary of the support domain will not cut across the interface and be limited by it. Also, the nodes on the border, will share a part of the domains on each side. The results show good accuracy. It was also seen that error norm depends only on the support domain.

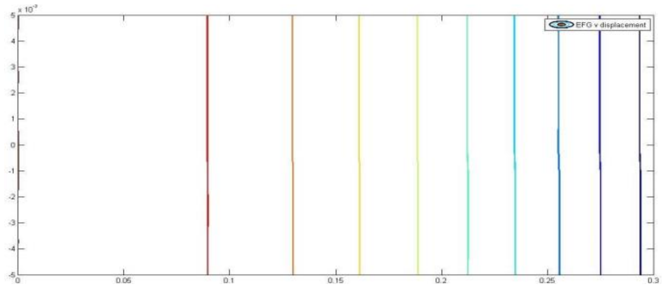

Fig. 5.3. The displacement contour for laminate plate

5.1. ANALYSIS OF THINPLATES

The isotropic plate considered is the one used in Belinha et al (2006). The details of the plate, geometry, loading etc. are as follows.

| Specifications | |
|---|---|
| E1=E2 (Gpa) | 30 |
| Poisson's ratio | 0.3 |
| G12 (Gpa) | 11.538 |
| q (kN/m$^2$) | 25 (udl) |
| Geometry | |
| L=b | 20 m |
| depth | 1m |
| Simply supported all sides | |

Table 5.2. Specifications for the isotropic plate

The implementation of EFGM for the isotropic plate involved the parametric study for the method to study the effect of different parameters like polynomial basis, weight function, quadrature order, support domain size, discretization, integration cell size, etc. The dependence of each of the parameter on other was studied. The results for the parametric study are presented here first.

## PARAMETRIC STUDY

Four types of weight functions were used in this work as described in Chapter 4. To understand the influence of the weight function on the result, the EFGM routine was run for different weight functions, other things like basis (linear), mesh size (10x10), integration cell mesh (10x10), were left unchanged. For each weight function, the 'd' parameter was kept changing and the variation in result with 'd' was noted. The results are given in the figures given below.

  

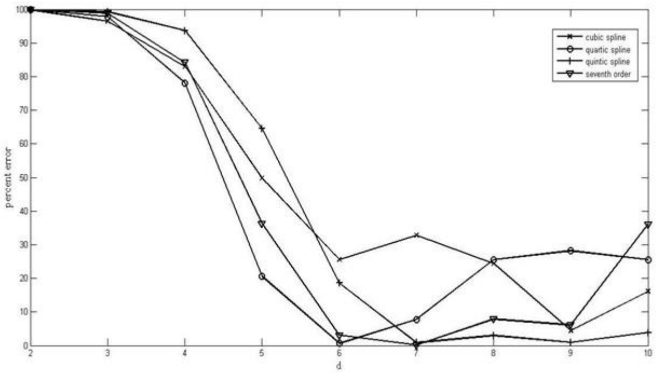


Fig 5.4. Error in percentage vs 'd' parameter for different weight function

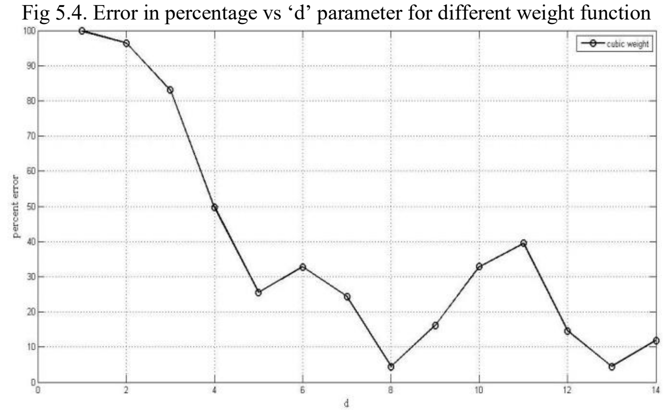


Fig. 5.5. Error in percentage vs 'd' for cubic weight case.

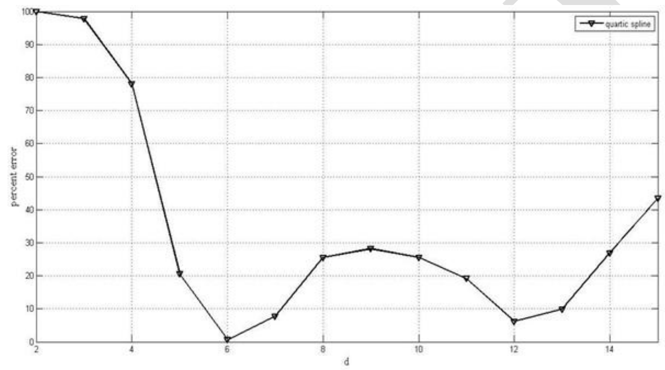


Fig. 5.6. Error in percentage vs 'd' parameter for quartic spline

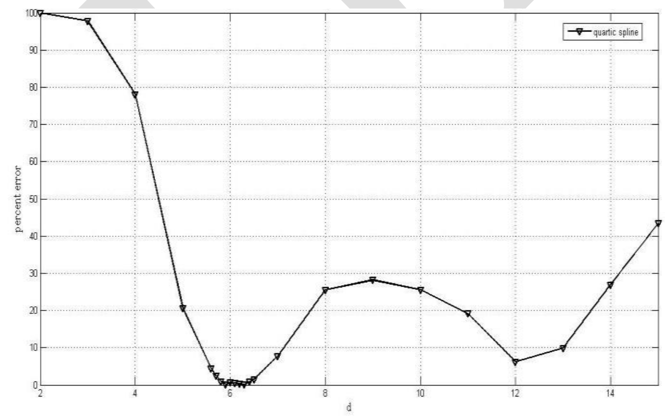


Fig. 5.7. Replot of Fig 5.5 with more points in the prospective optimal region

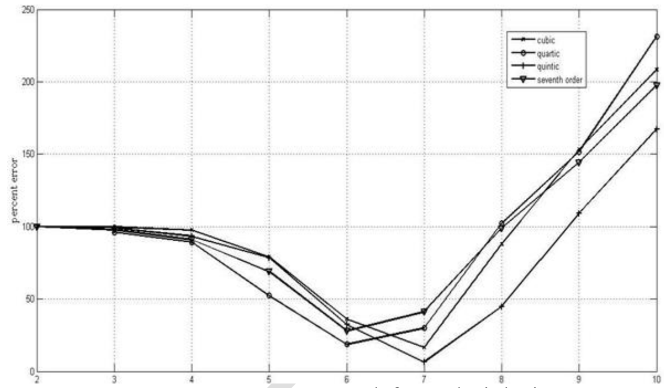


Fig. 5.8. Percent error vs 'd' for quadratic basis

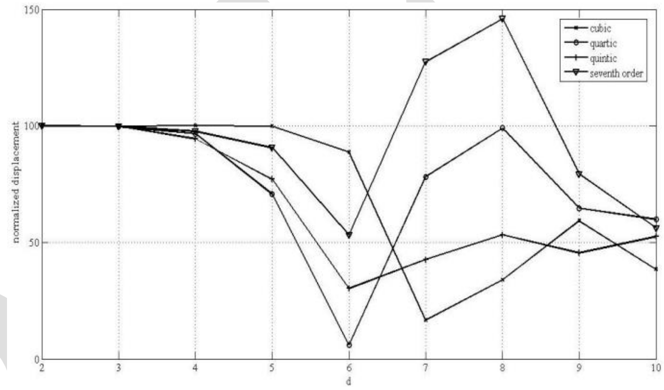


Fig 5.9. Percent error vs 'd' for cubic basis

The graphs presented above clearly show that linear basis and quintic spline function are the best options for the case of isotropic plate. It can be seen that the higher bases interestingly deliver poor results. This may be due to the fact that higher the order, more the optimal'd' would be, since more points are required to be within the domain. It can also be seen that even in the worst case, quintic weight performs better than other functions. Most importantly, quintic weight gives the most stable variation of result with 'd', that is the result almost converges after reaching the minimum 'd' required to obtain near exact result. This can't be said of the other functions which fluctuate widely even after obtaining the near exact results once. Especially, the seventh order spline function performs poorly. This may be due to the fact that the first and second derivatives for seventh order spline are singular, which is strictly prohibited for thin plates and laminates (Chen X.L., 2003). This is the reason why the quintic spline weight function performs well but seventh order spline, though costs more time, fails to provide a consistent result. Hence the choice of the weight function is crucial for the efficiency of EFGM algorithm. The weight function should suit the continuity requirement of the plate formulation. Fig. 5.5 shows the complex relationship between the 'd' and accuracy of result. The error value does not vary monotonously with the 'd'. Rather there is a lot of fluctuation of the result with variation in the 'd'. This is counter intuitive as one expects the d to increase monotonously with 'd' as it does for lower 'd' values. Also worth noting is the fact that the result varies even when 'd' is unrealistically large. In Fig. 5.5, the d values above 10 are meaningless since the number of elements in a direction is 10 and any increase of ‘d’ above it is not going to add any nodes to the domain of a point. Still, we can see that the result varies a lot. The reason for this strange behavior is that, though the number of nodes in the domain is same after a ‘d’ of 10, the weight function value at a given node varies. This is because the weight function is defined for the ratio of distance of a node from a point and the support size (i.e. ‘d’). This change in the value of weights assigned to the nodes influence the numerical integration result and also the MLS shape functions. Fig. 5.6 and 5.7 show that if the ‘d’ is varied even in the range of second decimal point, change in result can be obtained. Thus it is obvious the support size is very crucial to the functioning of the EFGM. Also it is obvious that the polynomial basis, the choice of weight function etc. influence heavily the variation of result with ‘d’. It is also obvious that since the curves do not converge except a few cases, finding an optimal ‘d’ for a problem where we do not know the exact result may be extremely difficult.

## RESULTS FOR ISOTROPICPLATE

The result for the isotropic plate is validated with the exact analytical results provided by Timoshenko [1959].

| Method used | Central Transverse Displacement(mm) |
|---|---|
| Exact ( Timoshenko) | 5.91126 |
| EFG- linear basis, cubic weight | 6.17410 |
| EFG- linear basis, quartic weight | 5.94995 |
| EFG- linear basis, quintic weight | 5.96184 |
| EFG- linear basis, seventh order weight | 5.90258 |

Table 5.3 Results for isotropic plate for various weight functions.

The results show that the current EFGM implementation has excellent compliance with the exact result. The result for linear basis has been presented as it was the best among the different bases used.

| Mesh | Integration cells | Central Transverse Displacement(mm) |
|---|---|---|
| 10 x 10 | 10 x 10 | 5.96184 |
| 20 x 20 | 10 x 10 | 5.96184 |

Table 5.4. Results for change in node density

| Gauss Points | Central Transverse Displacement(mm) |
|---|---|
| 4 x 4 | 6.17410 |
| 6 x 6 | 6.17199 |
| 8 x 8 | 6.17092 |
| 9 x 9 | 6.16646 |

Table 5.5. Results for change in quadrature rule

The tables 5.4 and 5.5 show that the use of a very fine mesh or a higher order quadrature rule alone does not guarantee better results. In fact, the improvement in the result is negligible in both cases. This indicates that the choice of a suitable basis and weight function for the problem in hand is far more important than discretization. The same behavior was observed for other bases and weight functions, the results of which have not been presented here. It was also observed that changing the mesh discretization caused slight or sometimes severe changes in the optimal support size. Also it was observed that placing the integration cells over the mesh is far more convenient than both being of different density, in which case two different support sizes have to be defined, further complicating the process. Hence, regular mesh and coinciding nodal mesh and integration mesh are found to be convenient.

| Specifications | |
|---|---|
| E1 (Gpa) | 250 |
| E2 (Gpa) | 10 |
| ν12 | 0.25 |
| ν21 | 0.01 |
| G12 (Gpa) | 11.538 |
| q (kN/m$^2$) | 100 |

| Geometry | |
|---|---|
| L=B | 20 m |
| D | 0.2 m |
| Simply supported all sides | |

Table 5.6. Specifications for laminate plate

The formulation for laminates is slightly different from that of isotropic plates. Most importantly, the presence of curvature terms in the Classical laminate theory formulation makes it necessary to have a higher order of continuity. Also it is observed that this behavior along with other factors that distinguish this problem from the isotropic plate problem causes changes to the effect the parameters have on the result. Hence a detailed parametric study is conducted for the laminate static analysis problem. The results of the same are presented in this section. The problem statement is as follows

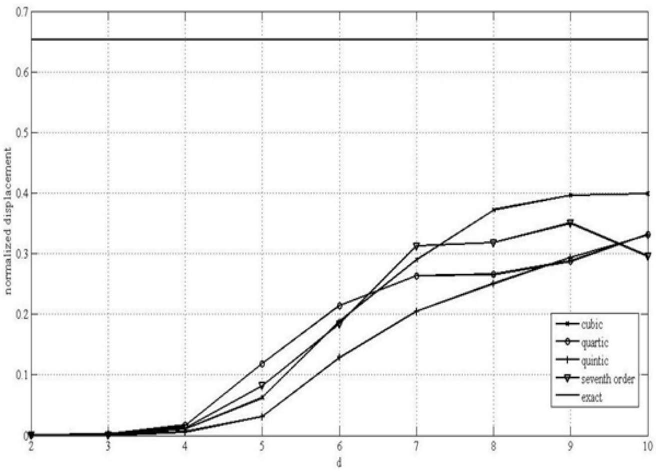


Fig.5.10. Effect of ‘d’ for linear basis

The plate specifications are as given in Belinha et al (2006). The parametric study and the results for laminates are discussed herby. The laminate is analyzed for different lay up schemes. The lay up schemes are as given in Belinha et al (2006). The results are validated with the exact results as per

  

Reddy [33] presented in Belinha et al (2006).

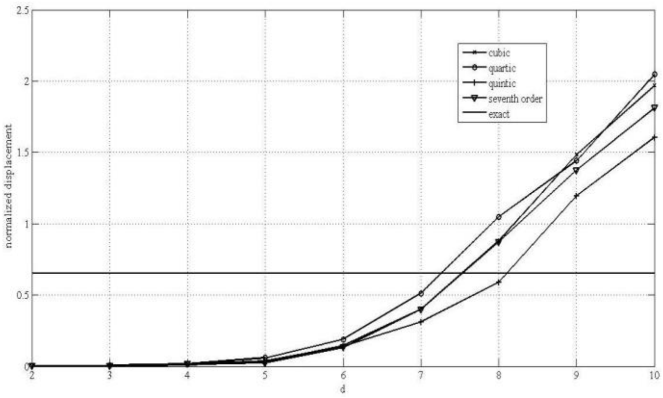


Fig. 5.11. Effect of ‘d’ for quadratic basis

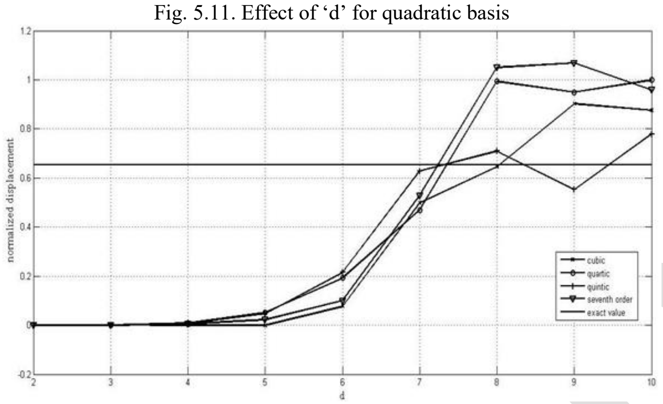


Fig. 5.12. Effect of ‘d’ for cubic basis.

The results in these graphs are for the one layer scheme. It can be observed that the linear basis which worked best for the thin plates performs badly for laminates. As discussed earlier, this is due to the fact that there are higher order derivatives of shape function involved and the shape function has to be C2 continuous. The quadratic basis also behaves the same way and fails to give a good result, though it performs better than linear basis and is theoretically sufficient. The cubic basis performs best among the three and gives excellent results. Especially the combination of cubic basis and quintic function works very well and is stable versus the support size change. As in the case of the plates, the increase in ‘d’ does not always cause monotonous change in results. There seems to be no clear relation between the ‘d’ and the displacement arrived at by EFGM Similar results have been obtained for other laminate stacking sequence too. The results of these are presented below

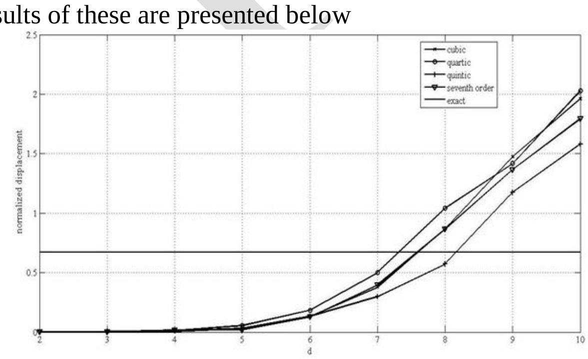


Fig. 5.13. Effect of ‘d’ for quadratic basis for 3 layer sequence

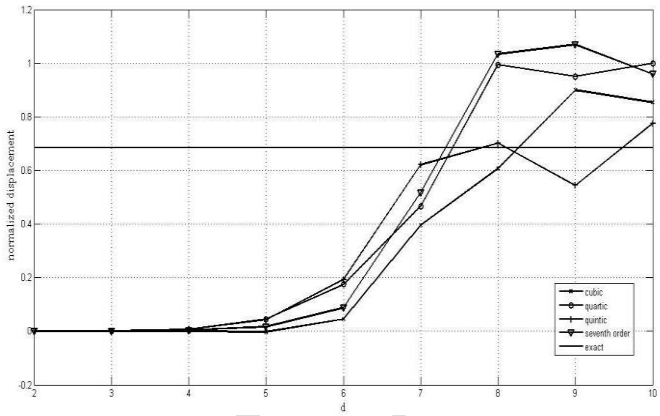


Fig. 5.14. Effect of ‘d’ for cubic basis and 3 layer sequence

The figures 5.13 and 5.14 are similar to those obtained for the one layer scheme. This denotes that the plate stacking sequence does not have an impact on the optimal support size parameter. This is significant because once an optimal ‘d’ value is found for a laminate of particular specifications, the result can be applied to any laminate sequence of similar specifications. The graphs also show that the seventh order function as applied by Belinha et al (2006) does not give consistent and accurate results as suggested in their work. It may be reasoned that such a weight function may be useful in some areas like elimination of shear locking etc., but it shows poorer performance the quantic function.

## RESULTS FOR STATIC ANALYSIS BYEFGM

The central transverse deflections for various cases of stacking sequences are presented here. The EFGM results match well with the analytical results.

| | Exact | Present EFG |
|---|---|---|
| $0^0$ | 0.6528 | 0.6465 |
| $0^0/90^0/0^0$ | 0.6697 | 0.6074 |
| $0^0/90^0/90^0/0^0$ | 0.6833 | 0.6923 |
| $0^0/90^0/0^0/90^0/0^0$ | 0.6874 | 0.6878 |
| $0^0/90^0/90^0/0^0/90^0/90^0/0^0$ | 0.6896 | 0.6835 |

Table 5.7. Results for optimal ‘d’=8 for different lay up schemes for h/t=1/100

It is obvious that the current formulation gives excellent results for all the cases. It must be noted that the optimal ‘d’ was taken as 8. However if we take upto two decimal points and then find the optimal ‘d’, the result would be still be close to the exact results. For example for the 1 layer scheme and the ‘d’ of 8.3, the result obtained is 0.6521 which is very close to the exact result. Hence with little more effort in choosing optimal ‘d’, still closer results could be achieved. Since choosing such a subtle parameter may not be convenient always, table 5.7 provides results for a ‘d’ of 8. Since the current formulation was based on computationally efficient Kirchoff’s plate theory, it seems to be successful in matching theoretical results while costing lesser computer time.

  

## CONCLUSION

The conclusions derived from current work are summarized as follows-

1. The EFGM provides reasonably accurate results for all the problems considered- plane stress, plate bending, and laminate bending.
2. The parameters of EFGM play a crucial role in determining the efficiency of the algorithm. The present work analyzed the major trends of variation caused by these The basis function and the weight function, which influence the trial and test function, are the most important parameters which affect the result. They have to be chosen in accordance to the theory employed for analysis.
3. The laminates needed a larger domain than the plate problem which in turn needs larger domain than the plane stress problem. This is due to the order of approximation required in each case. Higher the order, more the minimum number of neighboring nodes required to obtain decent approximations.
4. The error in approximation does not simply decrease with the increase in‘d’. The relation between these two is somewhere arbitrary.

## FUTURE SCOPE OF RESEARCH

1. The optimal ‘d’ parameter must be studied in depth to find possible ways to find the optimal parameter value even when one does not know the exact result for comparison.
2. The application of higher order laminate theories can extend the formulation presented to thicker plates. The formulation can also be extended to stiffened plates.
3. Dynamic analysis and 3D analysis can be conducted on laminates as an extension of current work.
4. Problems like stress concentration, delaminating study, etc can be conducted on the composites to lead to further studies in crack propagation.

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- Apply the meshfree methods for plane stress and plate bending problems in both isotropic plates and laminates. For plate bending, the theory used is thin plate theory and for composite, the analogous Classical Laminate theory is applied.
- The EFGM is dependent on the parameters involved in them, a detailed parametric study is also undertaken to facilitate the understanding of the influence of the parameters on the result.
- Interpretation of result and comparison with literature to assess the validity of the plate theories and performance of the meshfree approaches considered in the work.



  